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ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains a summary of problems in the film "Rotations in the Plane," and the supplement. (MK)

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THE ARITHMETIC PROJECT COURSE FOR TEACHERS

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FILM: Rotations in the Plane, Grade 5

SUPPLEMENT: Hybrid Rules: Jumping Rules from the Line to the Plane and from the Plane to the Line

20

NAME: .....

SE 031 120

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BOOK TWENTY

Summary of Problems in the Film "Rotations in the Plane"

5th Grade, Seldon L. Brown School, Wellesley, Massachusetts
Teacher: David A. Page

Here's a rule: $(\square, \triangle) \longrightarrow (-\triangle, \square)$

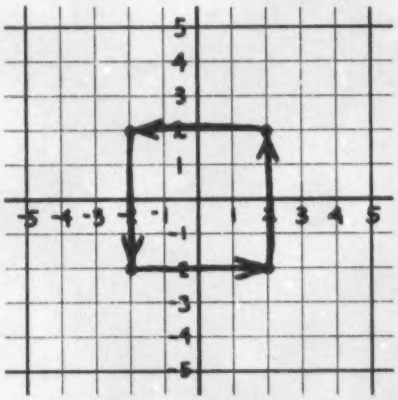
Start at $(2,0)$. Where do you land? $(0,2)$

Predict where we go next. $(-2,0)$, $(0,-2)$, $(2,0)$

Start at $(2,2)$. Where do you land? $(-2,2)$

Next jump? Another? Next?

Does anybody know a standstill point for this rule? $(0,0)$

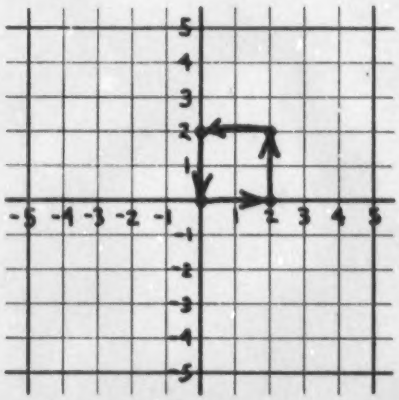


New rule: $(\square, \triangle) \longrightarrow (2 - \triangle, \square)$

Start at $(2,2)$ again. Where do you land after the first jump? $(0,2)$

Second jump? Third? Fourth?

What is the standstill point? $(1,1)$



Here's another rule: $(\square, \triangle) \longrightarrow \left(\frac{2 + \square - \triangle}{2}, \frac{\square + \triangle}{2}\right)$

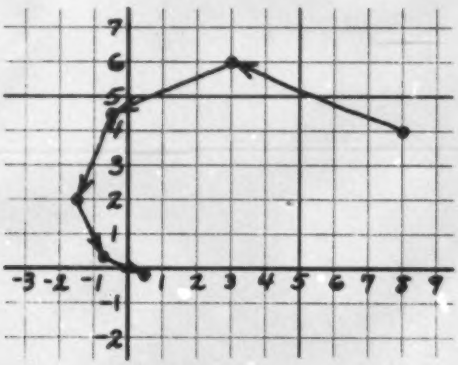
Start at $(8, 4)$. Land? $(3, 6)$

From $(3, 6)$ where do we go?

From $(-\frac{1}{2}, 4\frac{1}{2})$?

From $(-1\frac{1}{2}, 2)$?

From $(-\frac{3}{4}, \frac{1}{4})$? $(\frac{1}{2}, -\frac{1}{4})$



One of the questions we're trying to figure out is what will happen if we keep on making jumps? You can also be thinking about the standstill point. Any predictions as to what is going to happen?

(The jumps will become shorter.)

(The jumps will stop at the standstill point.)

Continue with the same rule.

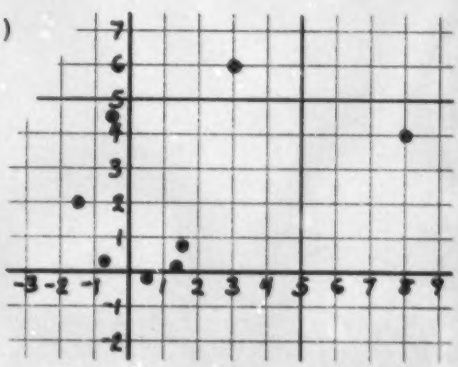
Start at $(\frac{1}{2}, -\frac{1}{4})$, where we left off.

Land? $(1\frac{3}{8}, \frac{1}{8})$

Who knows the standstill point? $(1, 1)$

Start at $(1\frac{3}{8}, \frac{1}{8})$. Land?

Will we ever actually hit the standstill point? What do you call something that has this shape?



Here is a problem for you to think about. How could you change this rule so that instead of the jumps getting closer and closer (spiraling in), they would get farther and farther away (spiraling out)? And how would you change the rule so you are balanced between a spiral in and a spiral out, so that the jumps go round and round? You can do that also by juggling the numbers.

Supplement

HYBRID RULES:
Jumping Rules From the Line to the Plane
and
From the Plane to the Line

by
Edward Esty

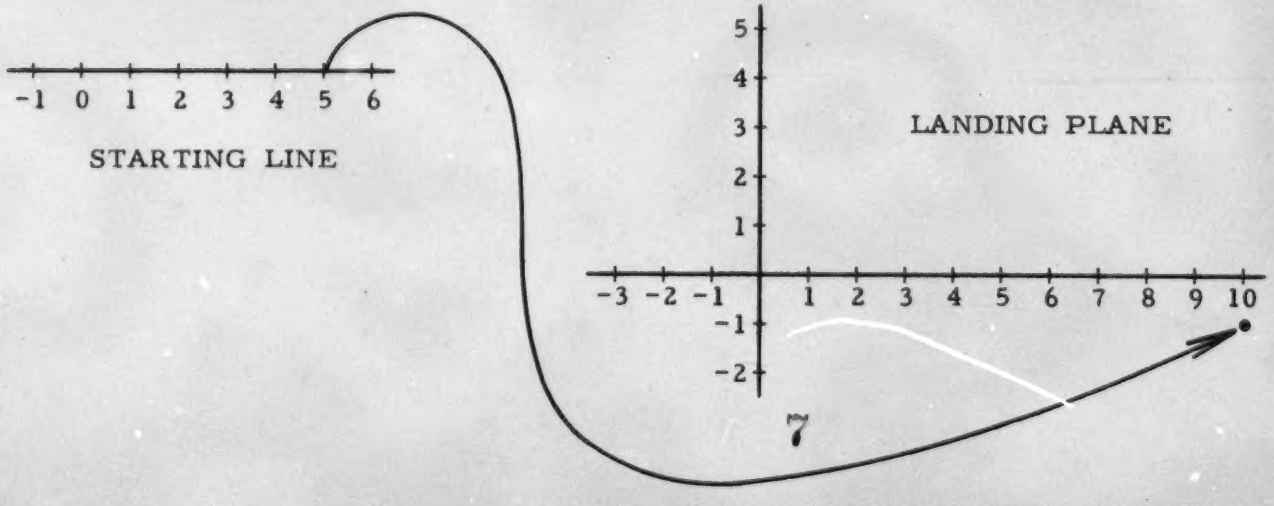
I.

By this time you are familiar with jumping rules from the line to the line (number line jumping rules) and from the plane to the plane (number plane jumping rules). Our subject here is two other kinds of jumping rules—those from the plane to the line and those from the line to the plane. We have already worked with one of these types (although under an entirely different name), as will become clear shortly.

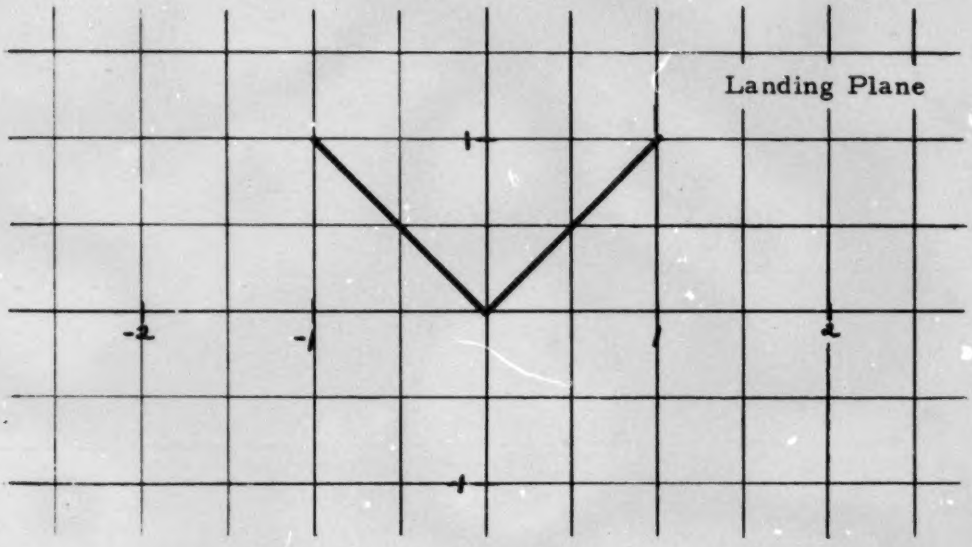
Here's an example of a jumping rule from the line to the plane:

$$\square \longrightarrow (2 \times \square, \square - 6)$$

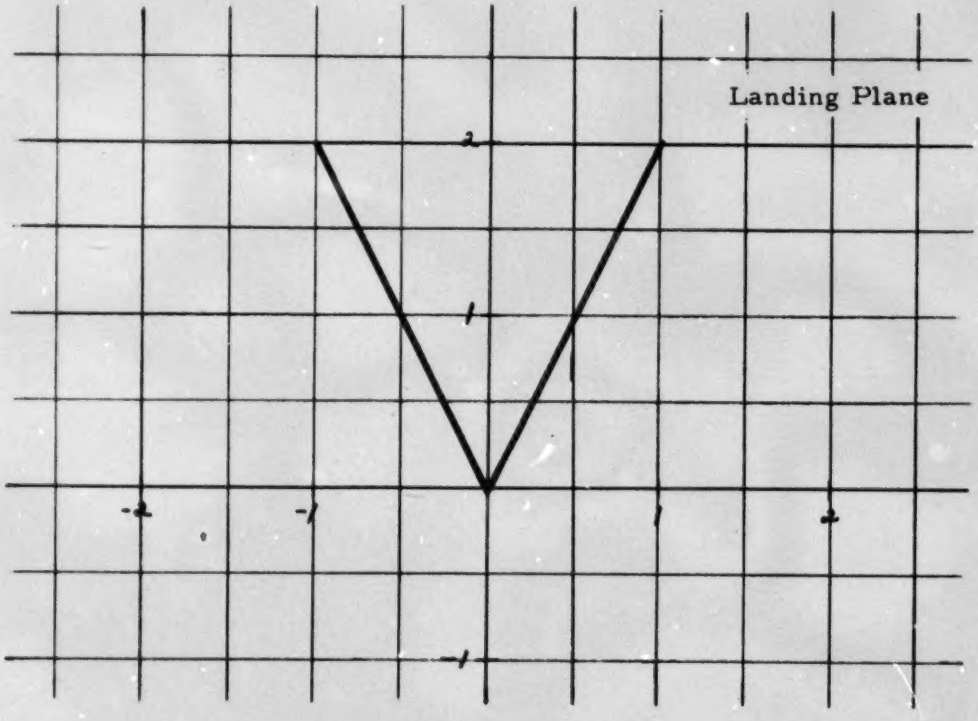
If we start at 5 (on the number line), we put 5 in the boxes and get $5 \longrightarrow (2 \times 5, 5 - 6)$ or $5 \longrightarrow (10, -1)$. Now $(10, -1)$ is an ordered pair—a point in the plane. So we have jumped from the line to the plane, and we might show the jump like this:



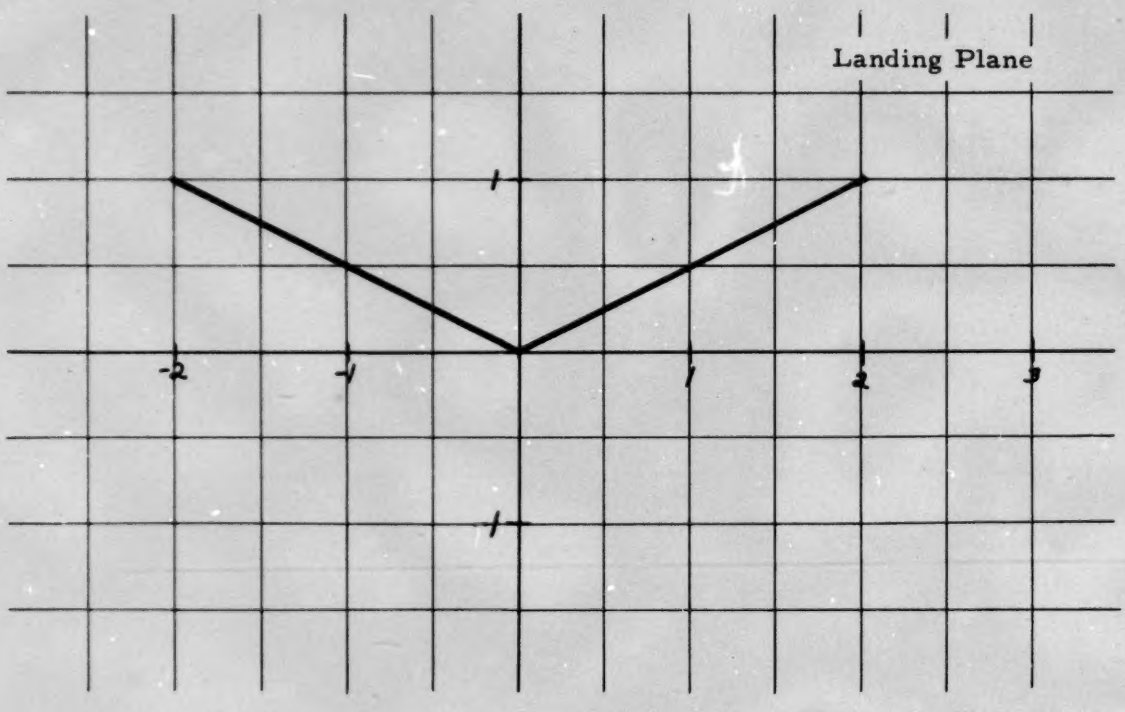
Consider the line-to-plane jumping rule $\square \longrightarrow (\square, |\square|)$. If we restrict our starting numbers to those between -1 and 1 inclusive, then our landing points are as shown below. Notice that this is <u>not</u> the graph of anything: the picture shows landing points, not jumps.

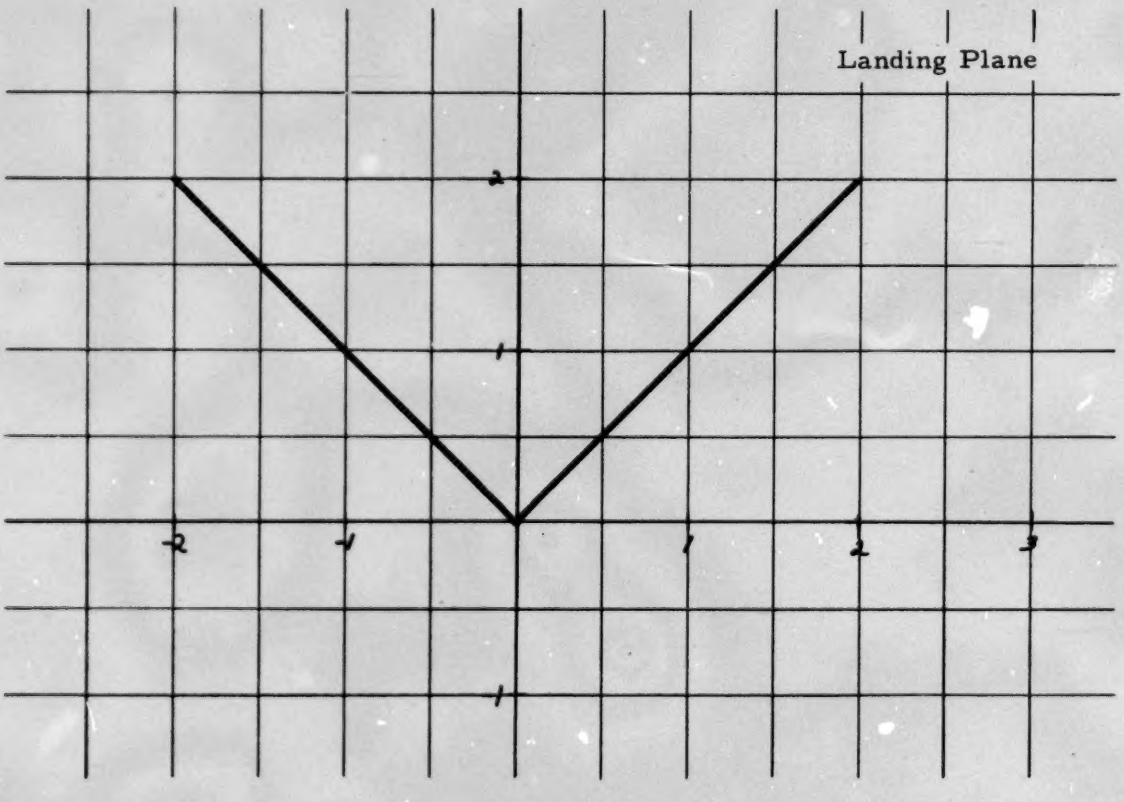


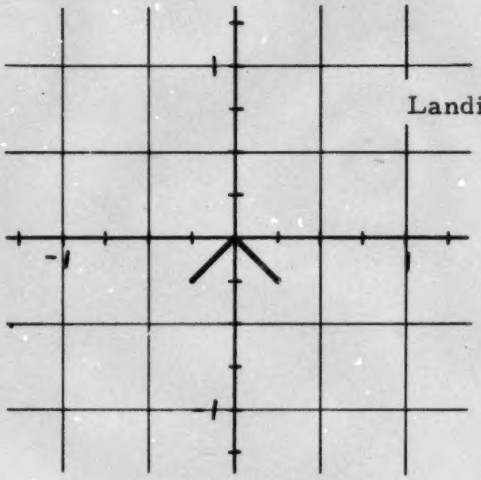
If we change the rule to $\square \longrightarrow (\square, 2 \times |\square|)$ and use the same starting numbers, our landing points will look like this:

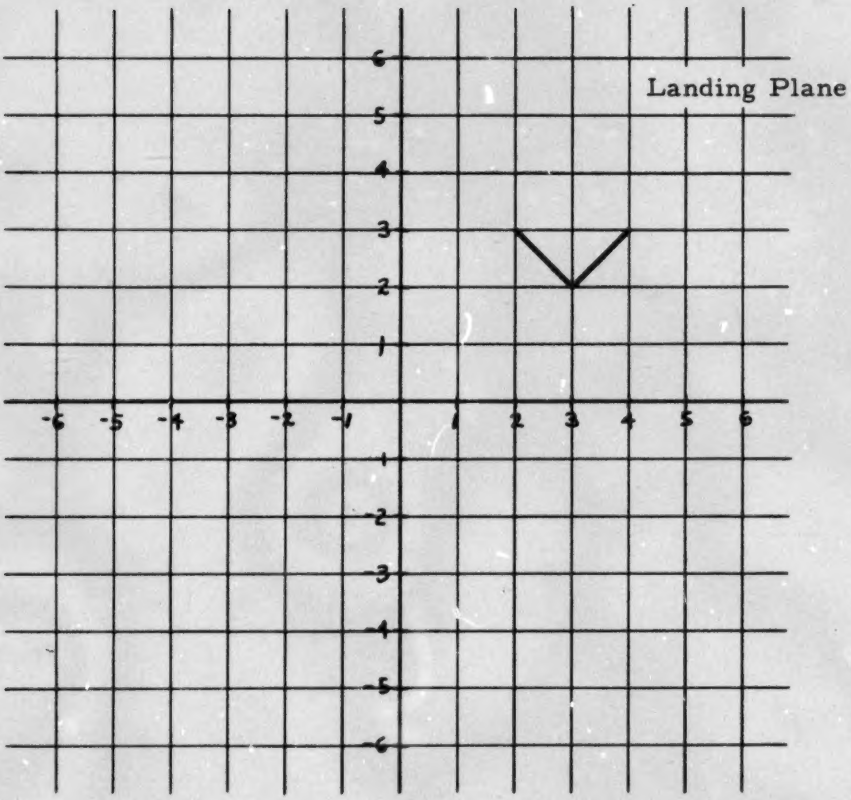


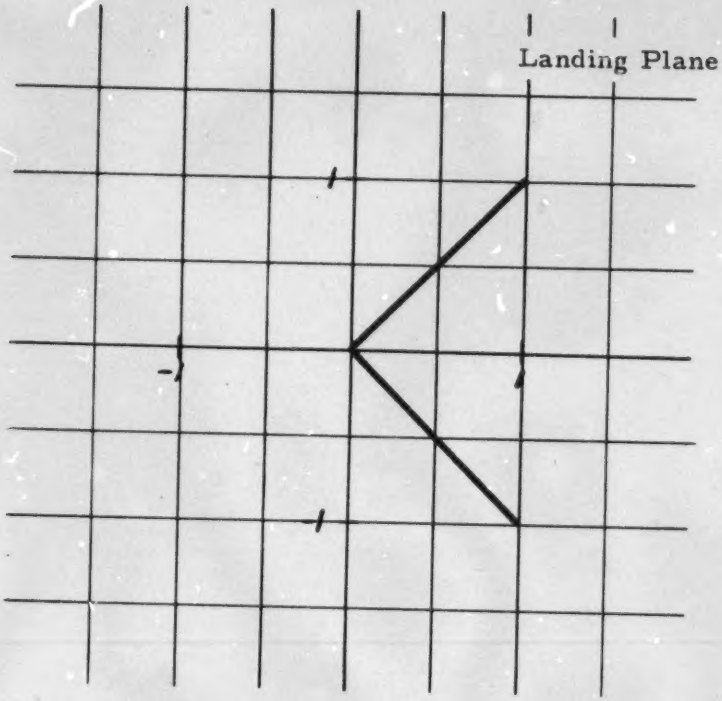
Using the same starting numbers how would you change the rule to get these landing points?

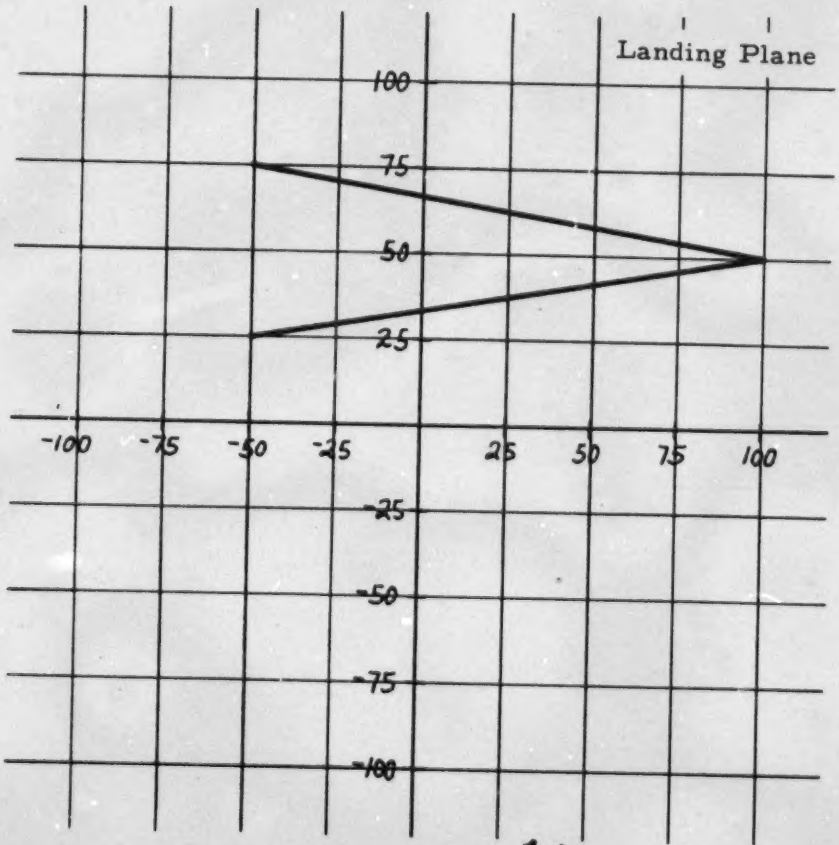





Landing Plane


Landing Plane


Landing Plane
1
-1
1
-1

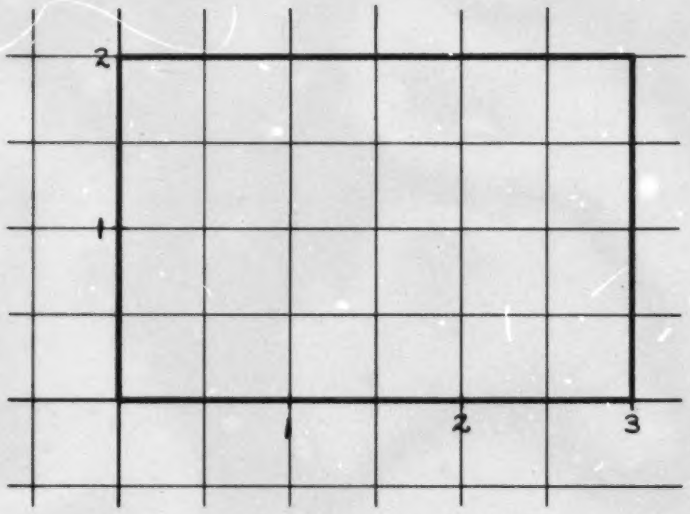

Landing Plane
100
75
50
25
-100
-75
-50
-25
25
50
75
100
-25
-50
-75
-100

8

Using somewhat fancier rules we can do fancier things. This time we restrict our starting numbers to those between 0 and 10 inclusive,† and use the rule:

$$\square \xrightarrow{A} \begin{cases} (\square, 0) & \text{if} \quad 0 \le \square \le 3 \\ (3, \square - 3) & \text{if} \quad 3 \le \square \le 5 \\ (8 - \square, 2) & \text{if} \quad 5 \le \square \le 8 \\ (0, 10 - \square) & \text{if} \quad 8 \le \square \le 10 \end{cases}$$

Our landing points then form a rectangle:



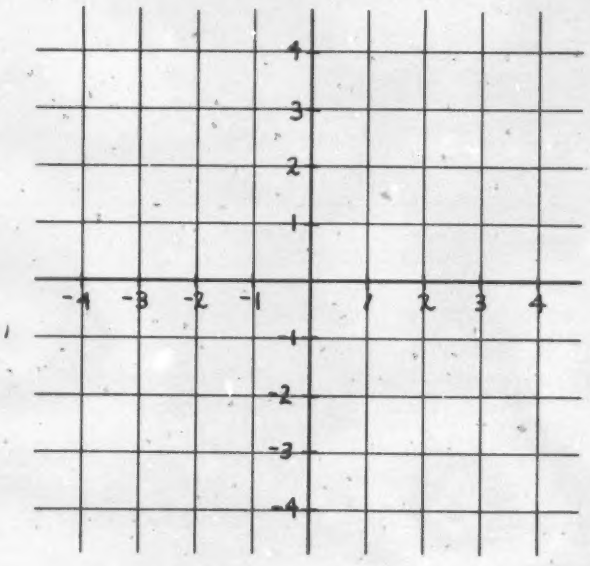
As before, we can change the rule to change the size, shape and position of the rectangle, but we assume that the reader can do this on his own. (It isn't so easy to change the rectangle into a triangle or a circle.)

† This interval of all numbers between 0 and 10, including 0 and including 10, is often written $\overline{0, 10}$.

One can say that rule A has the effect of wrapping the interval $\overline{0,10}$ around the rectangle. What would happen if we allowed other starting numbers; e.g., if the rule were this:

$$\square \xrightarrow{B} \begin{cases} (\square, 0) & \text{if} \quad \square \leq 3 \\ (3, \square - 3) & \text{if} \quad 3 \leq \square \leq 5 \\ (8 - \square, 2) & \text{if} \quad 5 \leq \square \leq 8 \\ (0, 10 - \square) & \text{if} \quad 8 \leq \square \end{cases}$$

(In rule B, all numbers on the number line are legal starting places.)



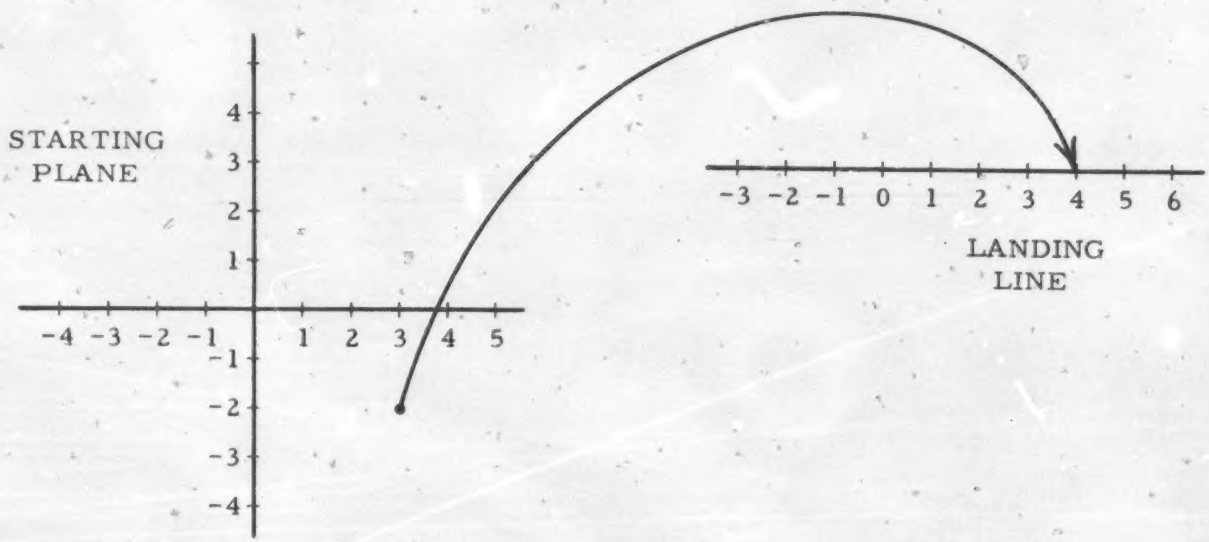
So rule B cannot be thought of as wrapping the entire number line around and around the rectangle. Can we change rule B so that we land on the rectangle no matter where we start? We will return later to this problem and to the problem of getting triangles and other figures.

II.

A jumping rule from the plane to the line must yield a single number when we start on an ordered pair of numbers. An example of such a rule is

$$(\square, \triangle) \longrightarrow 2 \times \square + \triangle$$

If we start on (3, -2) we put 3 in the boxes and -2 in the wedges. This gives us the jump (3, -2) $\longrightarrow$ 2 × 3 + -2 or (3, -2) $\longrightarrow$ 4 , which would be shown like this:



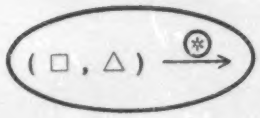
Notice that an ordered pair is two numbers in a specified order, so a plane-to-line jumping rule has the same effect as a binary operation: you put two numbers in and get one number out. For example, ordinary addition is given by the plane-to-line jumping rule ($\square$, $\triangle$) $\longrightarrow$ $\square + \triangle$. We will name such rules by putting a circle around the binary operation symbol, as follows:

$$(\square, \triangle) \xrightarrow{\oplus} \square + \triangle$$

$$(\square, \triangle) \xrightarrow{\otimes} \square \times \triangle$$

$$(\square, \triangle) \xrightarrow{\circledast} \square * \triangle$$

In general, then,



is another way of writing □ * △ , regardless of what the binary operation * is. Anything that can be done with binary operations can be done with plane-to-line rules.

Put a check in the correct column for each of these jumping rules:

Line to Line	Plane to Plane	Line to Plane	Plane to Line	None of These

(□ , △) ⟶ (3×□ , □ + △ + 2)

□ ⟶ (3×□ + □ + 2)

□ ⟶ (3×□ + △ + 2)

(□ , △) ⟶ 3 × (□ + △ + 2)

□ ⟶ (3×□ , □ + 2)

▽ ⟶ (3×▽ , ▽ + 2)

□ ⟶ (3×□ , △ + 2)

(□ , △) ⟶ (3×□ , □ + △ , 2)

(□ , △) ⟶ (0,0)

(□ , △) ⟶ 0

⊖

(△ , □) ⟶ (3×△ + □ , 3×□ + △)

(□ , △ , ▽) ⟶ □ + △ + ▽

III.

Composition of jumping rules of the foregoing kinds is a little trickier than composition of just number line rules because you must always keep in mind whether you are in the plane or on the line. Two plane-to-line rules cannot be composed, for example, since after using the first rule you would be on the line and thus you could not use the second rule. A plane-to-line rule can be followed by a line-to-line rule or by a line-to-plane rule, however. In the first case the new rule (obtained by the composition) would be a plane-to-line rule and in the second case a plane-to-plane rule.

Here are four rules:

$$\square \xrightarrow{a} 7\times\square + 100$$

$$(\square, \triangle) \xrightarrow{b} (7\times\square, \triangle + 100)$$

$$\square \xrightarrow{c} (7\times\square, \square + 100)$$

$$(\square, \triangle) \xrightarrow{d} 7\times\square + \triangle + 100$$

Put a check in the correct column for each of the rules listed below. The first two are done for you.

	Line to Line	Plane to Plane	Line to Plane	Plane to Line	Makes no Sense
ac			✓		
ca					✓
bbbbd					
cd					
cda					
dc					
dca					
aaaaacbb					
dcba					
b ⊕					
c ⊕⊕					

There are some jumping rules which are so frequently used that they are given special names. Since we know that you can resist the temptation to bring technical vocabulary prematurely into the classroom, we will list some of them here.

$\square \xrightarrow{i} (\square, 0)$	"i" stands for "imbedding". This jumping rule imbeds the number line in the plane as the first-number line.
$\square \xrightarrow{D} (\square, \square)$	"D" stands for "diagonal". If you make lots of jumps using this rule you'll see why it's called the diagonal jumping rule.
$(\square, \triangle) \xrightarrow{P_1} (\square, 0)$	"P" stands for "projection" and "1" stands for "first-number line".
$(\square, \triangle) \xrightarrow{P_2} (0, \triangle)$	Same idea here.
$(\square, \triangle) \xrightarrow{S} (\triangle, \square)$	"S" stands for "switching". (This is not an example of technical vocabulary.)

We need one further thing: If we have two number line jumping rules we can combine them to form a number plane jumping rule by having one number line rule operate on the first number and the other rule on the second number. If our two number line rules are f and g then we call the number plane rule f V g. By definition, then, f V g is the number plane rule given by

$$(\square, \triangle) \xrightarrow{f \vee g} (\square \xrightarrow{f}, \triangle \xrightarrow{g})$$

For example, if $\square \xrightarrow{f} \square + 1$ and $\square \xrightarrow{g} 2 \times \square$, then rule f V g is $(\square, \triangle) \xrightarrow{f \vee g} (\square + 1, 2 \times \triangle)$.*

*Many writers use the symbol $\times$ instead of V. We wish to avoid confusion with multiplication.

Continue with the rules:

$$\square \xrightarrow{f} \square + 1$$

$$\square \xrightarrow{g} 2 \times \square$$

Write out each of these rules explicitly and in as simple a way as you can. (We have put parentheses around each rule for clarity.)

$\square \xrightarrow{(D)(f \vee g)}$ $(\square + 1, 2 \times \square)$

$\square \xrightarrow{(i)(f \vee g)}$ _____

$\square \xrightarrow{(D)(f \vee g)(\oplus)}$ _____

$\square \xrightarrow{(D)(f \vee g)(P_1)}$ _____

$\square \xrightarrow{(D)(f \vee g)(P_1)(\oplus)}$ _____

$(\square, \triangle) \xrightarrow{(P_1)(\oplus)}$ _____

$(\square, \triangle) \xrightarrow{(P_1)(\oplus)(D)(f \vee g)(S)(P_2)(\oplus)}$ _____

$(\square, \triangle) \xrightarrow{(P_1)(\oplus)(D)(f \vee g)(S)(i)(\oplus)}$ _____

(Did you find the one problem above which makes no sense?)

V.

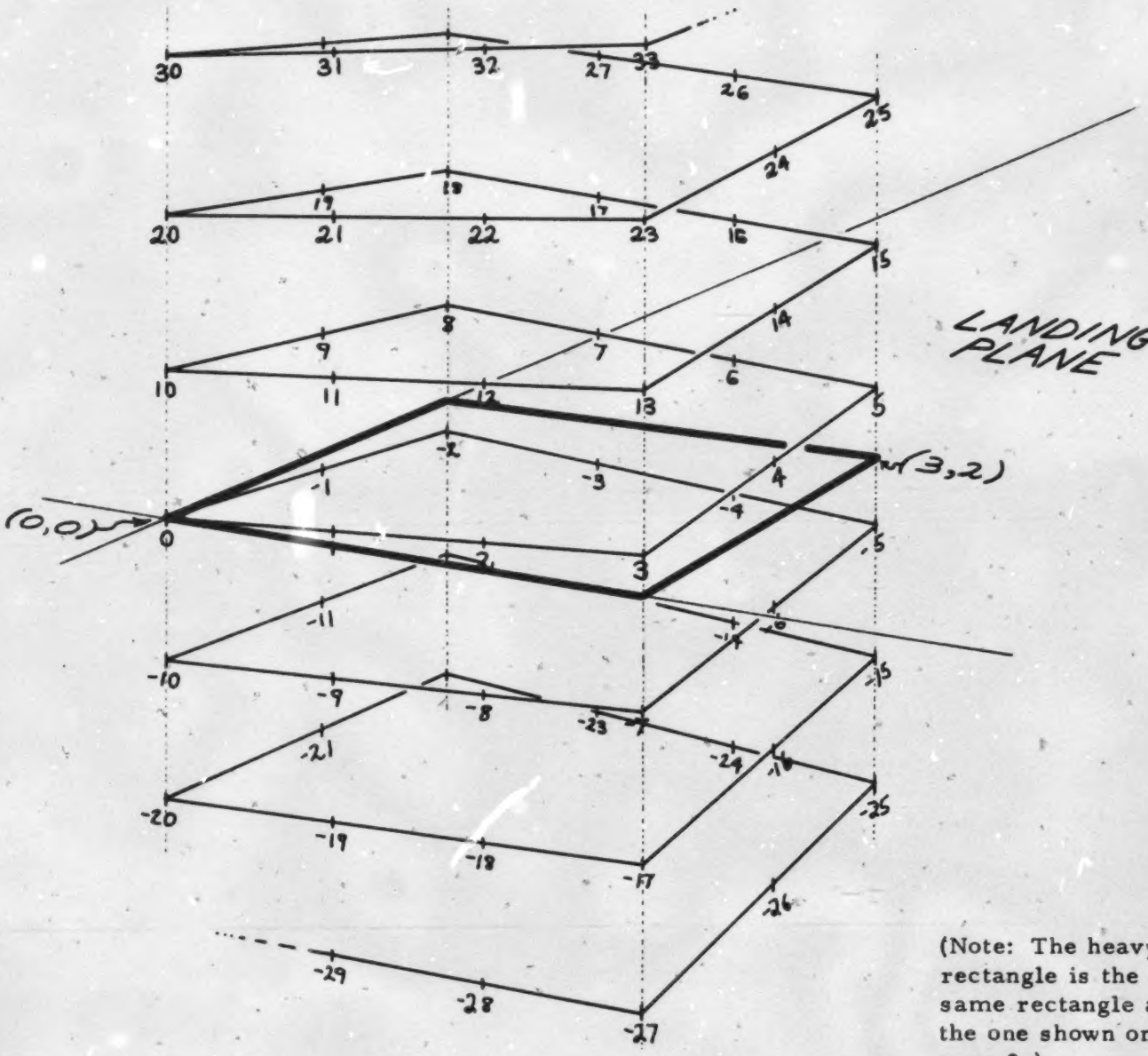
We return now to some of the problems presented in section I.

(1.) Wrapping the whole line around a rectangle.

If we can find a number line jumping rule which sends $94\frac{3}{4} \longrightarrow 4\frac{3}{4}$, $38 \longrightarrow 8$, $-\frac{1}{2} \longrightarrow 9\frac{1}{2}$, $1030 \longrightarrow 0$, etc.; i.e., which sends $\square$ to ($\square$ minus the largest multiple of 10 not larger than $\square$), then we can follow it by rule A (page 9) and hope that the composition will work. The number line rule we want is

$$\square \xrightarrow{t} \square - 10 \times \left\lfloor \frac{\square}{10} \right\rfloor$$

One way of visualizing rule tA is to imagine that the number line has been bent into a sort of square circular staircase, as the picture below shows:



(Note: The heavy rectangle is the same rectangle as the one shown on page 9.)

What tA does is to collapse this distorted number line onto the rectangle.

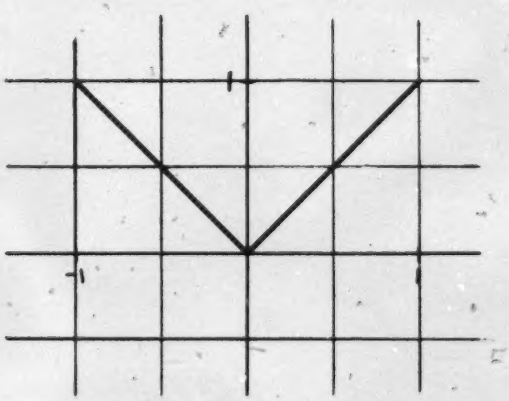
Would rule tB do the same thing?

An interesting twist is to follow rule tA with rule P_1, which squashes the rectangle flat onto the first number line. If we imagine a point moving along the starting number line then the image of that point using rule tAP_1 will be a point in the plane moving back and forth from (0, 0) to (3, 0), pausing at each endpoint before reversing direction. Following tAP_1 with $\oplus$, to get out of the plane and back onto the number line, gives the number line jumping rule $tAP_1\oplus$, which is well worth graphing (in the plane, with a starting-number line and a landing-number line).

Describe what the rule $tAP_2\oplus$ does.

(2.) Getting triangles as landing points.

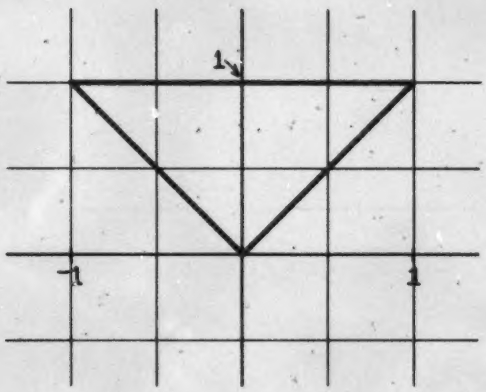
One way to do this is to close up an absolute value V-shaped figure by adding an extra line segment. We already know that the rule $\square \longrightarrow (\square - 1, |\square - 1|)$ takes the interval $\overline{0,2}$ into the figure below, since the rule $\square \longrightarrow (\square, |\square|)$ takes the interval $\overline{-1,1}$ into the same figure.



We extend our rule to allow numbers in the interval $\overline{0,4}$ and write

$$\square \xrightarrow{C} \begin{cases} (\square - 1, |\square - 1|) & \text{if} \quad 0 \le \square \le 2 \\ (3 - \square, 1) & \text{if} \quad 2 \le \square \le 4 \end{cases}$$

The second part of the rule puts the top on the triangle, and the landing points are shown here:

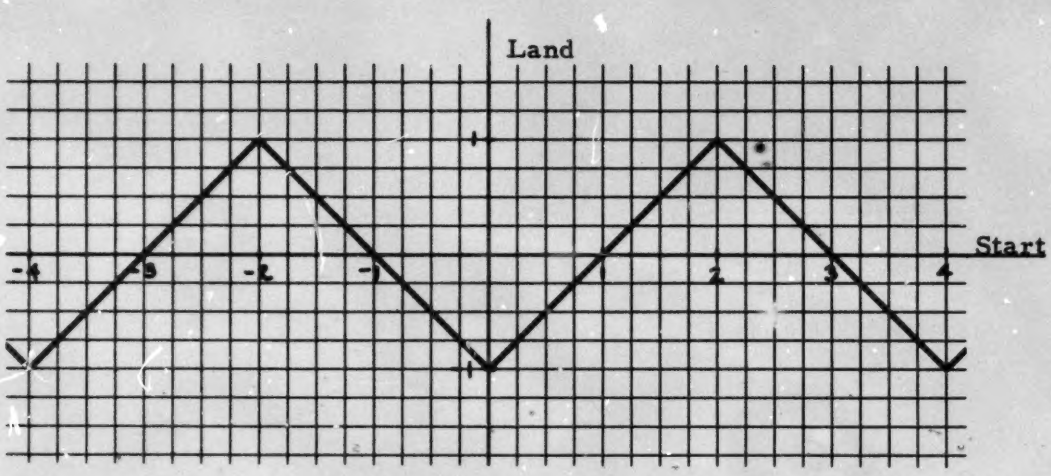


The interval $\overline{0,4}$ is wrapped around the triangle.

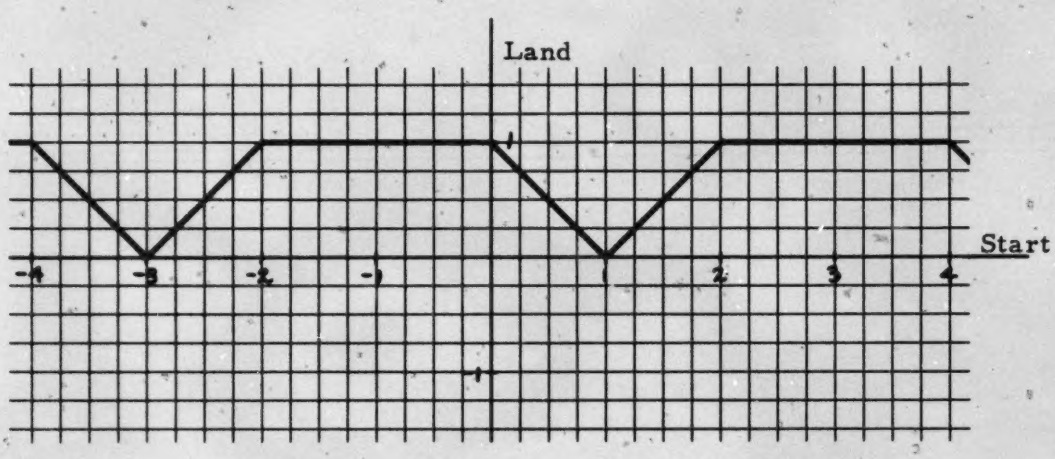
Again we could extend this rule so that the whole number line is wrapped around the triangle by preceding rule C with the rule

$$\square \xrightarrow{f} \square - 4 \times \left\lfloor \frac{\square}{4} \right\rfloor$$

And again we can squash the triangle down or sideways by following fC by P_1 or P_2, respectively, and then move from the plane to the line by using $\oplus$. The graphs of the number line jumping rules $fCP_1\oplus$ and $fCP_2\oplus$ are shown on the next page:



Graph of $fCP_1\oplus$

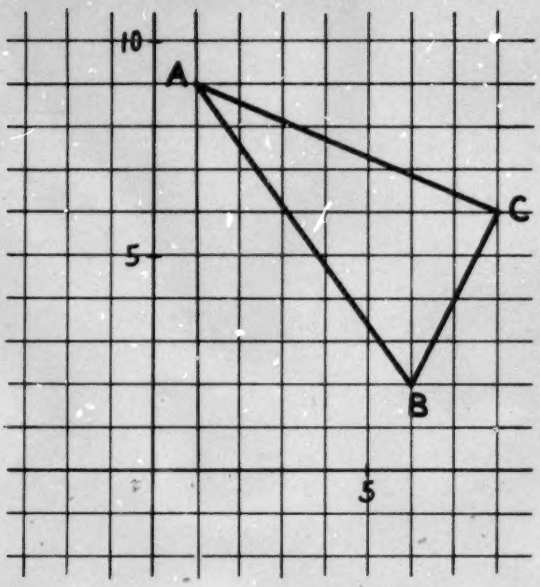


Graph of $fCP_2\oplus$

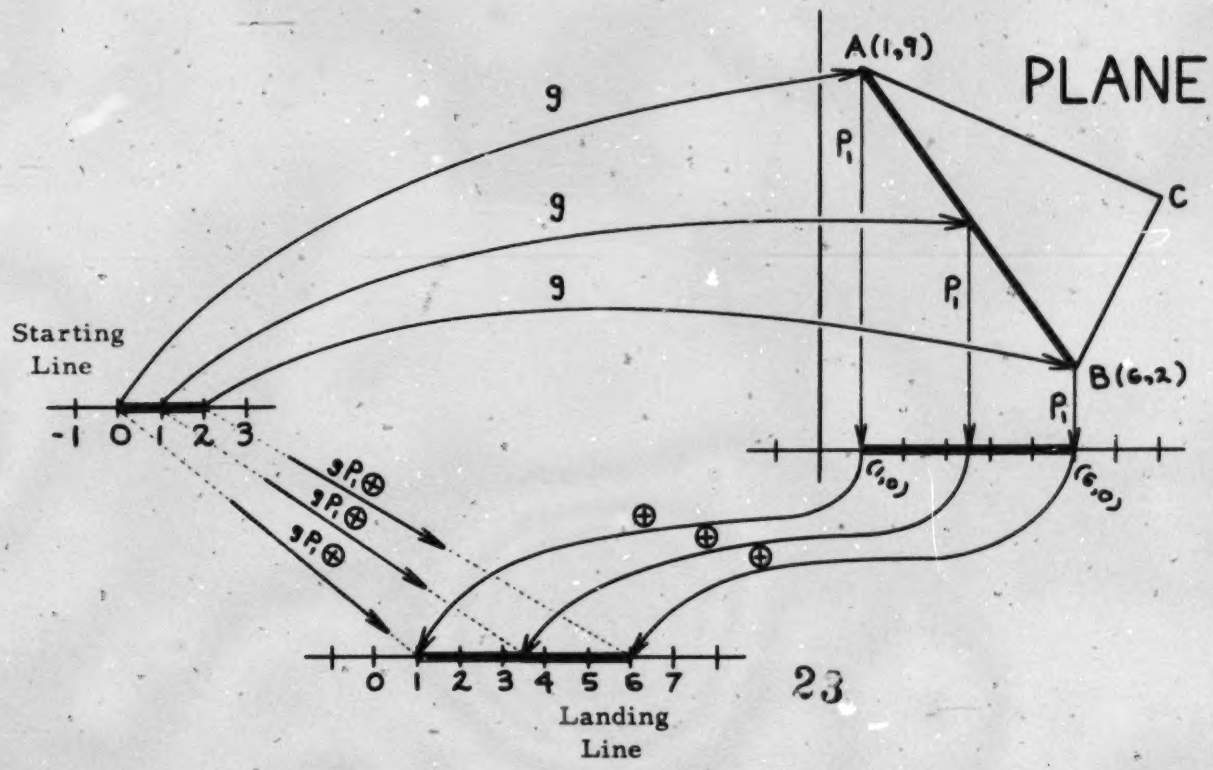
(These jumping rules are called <u>periodic</u> jumping rules for the same reason periodic lattices are called periodic.)

But what if the triangle cannot be built easily by using absolute value? We can break up our starting interval into three pieces and send each piece to one of the sides of the triangle, but only if we can tilt them. (No triangle has all three of its sides horizontal or vertical.)

Suppose our starting interval is $\overline{0,6}$ and we want our landing points to form the triangle shown at the right:



First we split $\overline{0,6}$ into $\overline{0,2}$, $\overline{2,4}$, and $\overline{4,6}$. We decide (arbitrarily) that we will send $\overline{0,2}$ onto the line segment AB, with 0 going to A and 2 to B. That is, we want a rule g, with $0 \xrightarrow{g} (1,9)$ and $2 \xrightarrow{g} (6,2)$. If we have such a rule, then $gP_1\oplus$ sends 0 to 1 and 2 to 6. If we can make $gP_1\oplus$ a linear rule, then $gP_1\oplus$ will send $\overline{0,2}$ to $\overline{1,6}$. The effects of rules g, P_1 and $\oplus$, and their composition, $gP_1\oplus$, are shown below:



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So rule $gP_1\oplus$ is a number line jumping rule, and finding it is just a two-points-to-two-points problem. We want $gP_1\oplus$ to take 0 to 1 and 2 to 6. The linear rule which does this is

$$\square \xrightarrow{gP_1\oplus} 2\tfrac{1}{2}\times\square + 1$$

Similarly we want $gP_2\oplus$ to be a linear rule sending 0 to 9 and 2 to 2. The rule

$$\square \xrightarrow{gP_2\oplus} -3\tfrac{1}{2}\times\square + 9$$

does this.

To form our final rule we first send $\square$ to $(\square,\square)$ via the diagonal rule D, and then use $gP_1\oplus$ on the first box and $gP_2\oplus$ on the second. So the rule g we want is $(D)\Big((gP_1\oplus)\vee(gP_2\oplus)\Big)$, or

$$\square \xrightarrow{g} \left(2\tfrac{1}{2}\times\square + 1\ ,\ -3\tfrac{1}{2}\times\square + 9\right)$$

Next we want a rule h which will send the interval $\overline{2,4}$ to the line segment BC, with 2 going to B and 4 to C—that is, we want $2 \longrightarrow (6,2)$ and $4 \longrightarrow (8,6)$. As before, we do the two two-points-to-two-points problems

$$\begin{array}{ll} & 2 \longrightarrow 6 \\ & 4 \longrightarrow 8 \end{array}$$

$$\begin{array}{ll} \text{and} & 2 \longrightarrow 2 \\ & 4 \longrightarrow 6 \end{array}.$$

The first is solved by $\square \longrightarrow \square + 4$ and the second by $\square \longrightarrow 2\times\square - 2$. So rule h, (which is the same as $(D)\Big((hP_1\oplus)\vee(hP_2\oplus)\Big)$, just as before) is

$$\square \xrightarrow{h} (\square + 4\ ,\ 2\times\square - 2).$$

We leave it to the reader to fill in the last line of the rule which takes $\overline{0,6}$ onto the triangle ABC .

$$\square \xrightarrow{k} \begin{cases} \left(2\frac{1}{2}\times\square + 1 \ , \ -3\frac{1}{2}\times\square + 9\right) & \text{if} \quad 0 \le \square \le 2 \\ (\square + 4 \ , \ 2\times\square - 2) & \text{if} \quad 2 \le \square \le 4 \\ \underline{\qquad\qquad\qquad\qquad} & \text{if} \quad 4 \le \square \le 6 \end{cases}$$

As a point moves along the number line from 0 to 6 its image under rule k will move once around the triangle, beginning and ending at point A . If the point on the number line moves at a constant speed, where is its image moving fastest? Most slowly? If we first use the rule $\square \xrightarrow{u} \square - 6\times\left\lfloor\frac{\square}{6}\right\rfloor$, what will the graphs of the rules $ukP_1\oplus$ and $ukP_2\oplus$ look like?

A Note on the Origins of the Arithmetic Project Course for Teachers

The University of Illinois Arithmetic Project began in 1958 when officials of the Carnegie Corporation of New York asked David A. Page if he would be interested in forming a project to pursue work he had been doing with elementary school children. Professor Page had been with the University of Illinois Committee on School Mathematics from nearly its beginning and was also one of the seven teachers of the Physical Science Study Committee high school physics course in its first trial year. He had become convinced that young children could do, and enjoyed doing, far richer mathematics than was—and is still—their normal fare.

Support from the Carnegie Corporation and the University of Illinois made possible a project which could attract to itself sensitive and creative teachers, mathematicians interested in the possibilities of teaching young children, university students, artists and others who could find or dream up mathematical ideas and develop them into forms that children could work with. Teachers welcomed the ideas enthusiastically in summer institutes supported by the National Science Foundation and the University of Illinois.

In 1963, Education Development Center (then Educational Services Incorporated) invited Page to bring the Project to Massachusetts. With a grant from the National Science Foundation the Project began to prepare and test materials that would enable teachers and prospective teachers to learn how to use the ideas successfully with their own students. Demonstration classes had early been found to be an effective communications medium, and the Project shot more than a million feet of film of classes being taught by staff members. Institutes were held for teachers in many communities using successive drafts of the written lessons, the films as they were completed, and in time the materials for leaders.

The course is the result of so many kinds of contributions from so many people that I can do little more than list their names on the next page with the Project's gratitude.

Acknowledgement should be made to the school systems which sponsored some of the early institutes. In Massachusetts these included the schools of Concord (Robert S. Ireland, Superintendent), Newton (Charles E. Brown, Superintendent), Wellesley (John B. Chaffee, Superintendent), Framingham (George P. King, Associate Superintendent), Chelmsford (Thomas L. Rivard, Superintendent), and Watertown (Francis A. Kelly and Raymond R. Delaney were superintendent at different times during the Project's close association with that town). In Illinois institutes were held in East Maine School District No. 63 (Hugh E. McGuigan, Superintendent); and of course, the Project had a warm relationship with the schools of Champaign and Urbana.

With the exception of the sections on composition, the early drafts of all the lessons were prepared by David Page. Acknowledgement is due Edward Esty for the material dealing with composition, to Edmund DeWan for the artwork and layout throughout, and to Catherine Crowell Mann for the Corrector's Guides. The present edition was produced by Phyllis Klein and Lisa Gregorian, with help from Sally Agro.

Special credit should go to two people without whom the course would not have come into being: Kevin Smith and Jerrold R. Zacharias. And the names of two others must be mentioned: Francis L. Friedman and Lorraine T. Page did not live to see the endeavor complete, but their influence on it was nevertheless incalculable.

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